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The selection of the design parameters of the aerodynamically stabilized nanosatellite of the CubeSat standard

Igor Belokonov[a], Ivan Timbai[a*]

[a]*Samara State Aerospace University, 34, Moskovskoye Shosse, Samara, Russia, 443086*

Abstract

The problem of selecting the design parameters of aerodynamically stabilized nanosatellite of the CubeSat standard is considered. The formulas for selecting the design parameters (static stability factor, length, longitudinal moment of inertia) of the aerodynamically stabilized nanosatellite of the CubeSat standard are obtained. At the low circular orbits the design parameters provides deviation of the longitudinal axis of the nanosatellite from the center of mass velocity vector that is less than acceptable with a given probability on a given height at the known errors of angular velocity for separation system.

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Keywords: *Nanosatellite; CubeSat; Angle of attack; Cumulative distribution functions; Design parameters.*

1. Introduction

The passive or combined (passive in combination with active) orientation system are often used to ensure the required orientation of small satellites. As you know, the design conditions such as uncontrolled motion of satellites can only be ensured at the design stage by selecting its design and ballistic parameters, as well as specifying limits of the angular velocity generated by the separation system.

In paper [1] the task of ensuring the aerodynamic stabilization of the nanosatellite of the CubeSat standard by deploying the solar panels at an angle to the longitudinal axis after separation from the adapter is considered.
In this paper we consider the problem of selecting the design parameters (static stability factor, length, longitudinal moment of inertia) of the aerodynamically stabilized nanosatellite of the CubeSat standard.

* Corresponding author. Tel.:+7-846-267-44-44;
E-mail address: nibelung63@gmail.com

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At the low circular orbits the design parameters provides deviation of the longitudinal axis of the nanosatellite from the center of mass velocity vector that is less than acceptable with a given probability on a given height at the known errors of angular velocity for separation system.

In paper [2] the analytical cumulative distribution functions of the maximum of nanosatellite angle of attack for Rayleigh and uniform distributions of the initial transverse angular velocity value for the nanosatellite planar angular motion along the circular orbit under the influence of the gravitational and aerodynamic moments are obtained. It is shown that for the nanosatellite separation along the velocity vector (the angle of attack initial value is small), and small spinning about the longitudinal axis the analytical cumulative distribution functions of the maximum angle of attack can be used with sufficient precision. These functions are obtained for the case of planar angular motion, and for the case of spatial angular motion. In this paper these analytic functions form the basis for the obtaining of formulas for the selection of nanosatellite design parameters.

2. The selecting of design parameters of aerodynamically stabilized nanosatellite

It was decided that the nanosatellite wrapping is free-molecule and the gas molecules collision is absolutely inelastic, the resultant of the aerodynamic force is applied to the nanosatellite geometric center. In this case, the aerodynamic drag force is determined by the nanosatellite area projected on a plane that is perpendicular to the flow velocity vector [3]. The restoring aerodynamic moment coefficient measured about the nanosatellite center of mass is determined by the formula:

$$m_\alpha(\alpha,\varphi) = -c_0 \overline{S}(\alpha,\varphi)\Delta\overline{x}\sin(\alpha),$$

where $c_o = 2.2$ is the drag force coefficient; $\Delta\overline{x} = \Delta x / l$ is the relative static stability margin, Δx - is the static stability margin (the distance measured from the center of mass to the geometric center of the nanosatellite), l is the nanosatellite length; $\widetilde{S}(\alpha,\varphi)$ is the nanosatellite area projected on a plane that is perpendicular to the flow velocity vector divided by the characteristic nanosatellite square, α is the spatial angle of attack (the angle between the longitudinal axis and nanosatellite mass center velocity vector), φ is the proper rotation angle (the angle between the angle of attack plane and the lateral axis that is perpendicular to the side).

The area of the nanosatellite of the CubeSat standard, which has the form of rectangular parallelepiped with equal sides of a base on the plane that is perpendicular to flow velocity vector divided by the characteristic nanosatellite square, is determined by the formula:

$$\widetilde{S}(\alpha,\varphi) = |\cos(\alpha)| + k\,|\sin(\alpha)| \cdot \left(|\sin(\varphi)| + |\cos(\varphi)|\right),$$

where k is the ratio of the one side surface area to the characteristic square.

For the analysis of angular motion of the nanosatellite in the case when the angular velocity of proper rotation is close to uniform the restoring aerodynamic moment coefficient can be averaged over the angle of proper rotation:

$$m_\alpha(\alpha) = -c_0 \Delta\overline{x}\sin\alpha\left(|\cos(\alpha)| + \frac{4k}{\pi}|\sin(\alpha)|\right).$$

For approximate analysis of motion parameters, the dependence of the restoring aerodynamic moment coefficient measured about the nanosatellite center of mass $m_\alpha(\alpha)$, averaged over the angle of proper rotation φ, with sufficient accuracy can be approximated by a sine function by the angle of attack:

$$m_\alpha(\alpha) = a_0 \sin(\alpha).$$

Then change of the angle of attack of a dynamically symmetric nanosatellite under the influence of gravitational and restoring aerodynamic moments that is moving in a circular orbit is described by the following equation [3]:

$$\ddot{\alpha} - a(H)\sin\alpha - c(H)\sin 2\alpha = 0, \tag{1}$$

where $a(H) = a_0 Slq(H)/J_n$ is the coefficient associated with aerodynamic restoring moment; J_n is the transverse moment of inertia; S is the nanosatellite characteristic square; $q(H) = V^2\rho(H)/2$ is the velocity head; V is the flight speed; H is the orbit altitude; $\rho(H)$ is the atmospheric density; $c(H) = 3(J_n - J_x)(\omega(H))^2/(2J_n)$ is the coefficient associated with the gravitational moment; J_x is the longitudinal moment of inertia; $\omega(H) = \sqrt{\mu/(R_E + H)^3}$ is the nanosatellite angular orbital velocity; R_E is the radius of the spherical Earth; μ is the Earth's gravitational parameter.

The value of the maximum angle of attack after the nanosatellite separation from the adapter is random and is determined by the initial value of the angle of attack α_0, the initial value of the angular velocity $\dot{\alpha}_0$, the aerodynamic and gravitational moments. Assuming that among these values the angular velocity has the largest range of values and neglecting the scatter of other variables, for the motion model (1) in [2] are obtained the cumulative distribution function of the maximum angle of attack $\alpha_{\max}$ at the time of separation from the adapter. If the module values $\dot{\alpha}_0$ are distributed according to the Rayleigh law, the cumulative distribution function of the maximum angle of attack will be determined by the formula:

$$F(\alpha_{\max}) = 1 - \exp\left(\frac{-a(\cos\alpha_{\max} - \cos\alpha_0) - c(\cos^2\alpha_{\max} - \cos^2\alpha_0)}{\sigma^2}\right), \tag{2}$$

where $\sigma > 0$ is scale parameter of the distribution.

If the module values $\dot{\alpha}_0$ are distributed according to a uniform law in the range $[0, \dot{\alpha}_{0\max}]$, the cumulative distribution function of the maximum angle of attack will be determined by the formula:

$$F(\alpha_{\max}) = \frac{\sqrt{2a(\cos\alpha_{\max} - \cos\alpha_0) + 2c(\cos^2\alpha_{\max} - \cos^2\alpha_0)}}{\dot{\alpha}_{0\max}}. \tag{3}$$

Specifying p^* (the probability of the maximum allowable angle of attack $\alpha^*_{\max}$), solving (2), (3) with respect to the coefficient associated with the aerodynamic restoring moment we obtain:
in the case of the distribution of the initial angular velocity $\dot{\alpha}_0$ according to the Rayleigh law

$$a = -\frac{\sigma^2\ln(1-p^*) + c(\cos^2\alpha^*_{\max} - \cos^2\alpha_0)}{\cos\alpha^*_{\max} - \cos\alpha_0}; \tag{4}$$

in the case of the distribution of the initial angular velocity $\dot{\alpha}_0$ according to a uniform law

$$a = \frac{\left(\dot{\alpha}_{0\max}p^*\right)^2 - 2c\left(\cos^2\alpha^*_{\max} - \cos^2\alpha_0\right)}{2\left(\cos\alpha^*_{\max} - \cos\alpha_0\right)}. \tag{5}$$

In order to solve equations (4) and (5) with respect to the nanosatellite design parameters, the coefficient of the approximation of the sinusoidal dependence on the angle of attack of the restoring aerodynamic moment coefficient measured about the nanosatellite center of mass can be approximately calculated as follows:

$$a_0 \approx m_\alpha(\pi/2) = -c_0\Delta\bar{x}\frac{4k}{\pi}. \tag{6}$$

For example, the coefficient a_0 calculated by least square method for the nanosatellite of the CubeSat 2U standard with dimensions 0.1 × 0.1 × 0.2 m^3 is equal to a_0 = -1.14, but using the formula (6) a_0 = -1.12, coefficient

a_0 calculated by least squares method for nanosatellite of the CubeSat 3U standard with dimensions 0.1 × 0.1 × 0.3 m^3 is equal to a_0 = -1.68, but using the formula (6) a_0 = -1.61, which are close enough and does not change the result of the research.

Substituting the expression for the coefficient associated with aerodynamic restoring moment a in (4), (5), taking into account (6), solving (4), (5) with respect to the design parameters, combined in a structural parameter, we obtain the requirement to its value. For the maximum angle of attack to be less than allowable value with a probability no less than p^* it is necessary to satisfy the following conditions for the nanosatellite structural parameter:
in the case of the distribution of the initial angular velocity according to the Rayleigh law

$$d = \frac{\Delta x}{J_n} lb \geq d_{r1} = \frac{\pi\left(\sigma^2 \ln(1-p^*) + c(\cos^2\alpha^*_{\max} - \cos\alpha_0)\right)}{4c_0\left(\cos\alpha^*_{\max} - \cos\alpha_0\right)q(H)}; \tag{7}$$

in the case of the distribution of the initial angular velocity according to a uniform law

$$d = \frac{\Delta x}{J_n} lb \geq d_{r1} = \frac{\pi\left(\left(\dot{\alpha}_{0\max} p^*\right)^2 - 2c\left(\cos^2\alpha^*_{\max} - \cos^2\alpha_0\right)\right)}{8c_0\left(\cos\alpha_0 - \cos\alpha^*_{\max}\right)q(H)}, \tag{8}$$

where b is a side of the base of the rectangular parallelepiped.

It should be noted that it is necessary also to satisfy the following condition: the restoring aerodynamic moment must exceed the gravitational moment and determine the nature of motion, i.e. it is necessary to satisfy the condition $|a| \geq 2|c|$ [3]. This condition, using the expressions for the coefficients a and c, can be written in the form of requirements for the design parameters:

$$d = \frac{\Delta x}{J_n} lb \geq d_{r2} = \frac{\pi c}{4c_0 q(H)}. \tag{9}$$

It should be noted that the requirement (9) is much weaker than the requirements (7) and (8) in a wide range of design parameters.

The expressions (7), (8), (9) include the coefficient c associated with the gravitational moment, which varies slightly over the height compared to the coefficient a, associated with restoring aerodynamic moment. Therefore calculating the value of the coefficient c at altitude H = 150 km and taking into account that under the condition $J_n > J_x$ the ratio of the difference of transverse and longitudinal to transverse moments of inertia can not exceed unity, we obtain the limit value of the coefficient $c = 2.2 \cdot 10^{-6}$ rad/s^2. This coefficient can be used for the upper bound of the required value of the structural parameter.

At altitudes where the restoring aerodynamic moment is much greater than the gravitational moment coefficient c can be neglected. Then the expressions (7) and (8) will take the form:

$$d = \frac{\Delta x}{J_n} lb \geq d_{r1} = \frac{\pi\sigma^2 \ln(1-p^*)}{4c_0\left(\cos\alpha^*_{\max} - \cos\alpha_0\right)q(H)}, \tag{10}$$

$$d = \frac{\Delta x}{J_n} lb \geq d_{r1} = \frac{\pi\left(\dot{\alpha}_{0\max} p^*\right)^2}{8c_0\left(\cos\alpha_0 - \cos\alpha^*_{\max}\right)q(H)}. \tag{11}$$

For small values of α_0 (separation of the nanosatellite is carried out along the velocity vector) the expression $\cos\alpha_0 = 1$ can be accepted. In this case the expressions (10) and (11) will take the form:

$$d = \frac{\Delta x}{J_n} lb \geq d_{r1} = \frac{\pi\sigma^2 \ln(1-p^*)}{4c_0\left(\cos\alpha^*_{\max}-1\right)q(H)}, \tag{12}$$

$$d = \frac{\Delta x}{J_n} lb \geq d_{r1} = \frac{\pi\left(\dot{\alpha}_{0\max} p^*\right)^2}{8c_0\left(1-\cos\alpha^*_{\max}\right)q(H)}. \tag{13}$$

Using the obtained expressions (7)-(13), we can construct the nomograms to assess the possibility of implementation of the required values of the structural parameter. For example, in Fig. 1, 2 on the right the dependencies of the required structural parameter of the nanosatellite on orbit with the altitude H and on the parameter σ (the initial transverse angular velocity is distributed according to the Rayleigh law) are shown: for the values of the maximum angle of attack $\alpha^*_{\max} = 20\,\deg$ (Fig. 1), $\alpha^*_{\max} = 30\,\deg$ (Fig. 2), the probability $p^* = 0.95$ and an initial angle of attack $\alpha_0 = 0$, and on the left values of the structural parameter of the nanosatellite of the CubeSat 3U ($0.1 \times 0.1 \times 0.3 m^3$) standard with different values of the transverse moment of inertia on the static stability margin and the structural parameter of the aerodynamically stabilized nanosatellite with the transformable design of the SamSat QB50, developed as the part of the international project QB50 [5], are shown. Calculations were performed for the standard density of the atmosphere in accordance with the standard GOST 4401-81. [4]. The SamSat QB50 nanosatellite of the transformable design is the 2 kg nanosatellite having an initial CubeSat 2U form with dimensions 0.1 × 0.1 × 0.2 m^3 and the initial distance between the center of pressure and center of mass $\Delta x = 0.02$ m. After separation from the adapter it transforms into nanosatellite CubeSat 3U form with dimensions of 0.1 × 0.1 × 0.3 m^3, thereby significantly increasing the distance between the center of pressure and center of mass (up to $\Delta x = 0.055$ m) [5].

In Fig. 3, 4 on the right the dependencies of the required structural parameter of the nanosatellite on orbit with the altitude H and on the value of the right border of interval for the initial transverse angular velocity $\dot{\alpha}_{0\max}$ (the initial transverse angular velocity is distributed according to a uniform law) are shown: for the values of the maximum angle of attack $\alpha^*_{\max} = 20\,\deg$, (Fig. 3), $\alpha^*_{\max} = 30\,\deg$ (Fig. 4), the probability $p^* = 0.95$ and the initial angle of attack $\alpha_0 = 0$.

In Fig. 5, 6 on the right the dependencies of the required structural parameter of the nanosatellite on orbit with the altitude H and on the value of given probability p^* are shown: for the values of the maximum angle of attack $\alpha^*_{\max} = 20\,\deg$, the initial angle of attack $\alpha_0 = 0$, the parameter $\sigma = 0.05\deg/s$ (Fig. 5) and the value $\dot{\alpha}_{0\max} = 0.15\deg/s$ (Fig. 6).

The nomograms can be used to select the design parameters of the nanosatellite, and for specifying the requirements to errors of the separation system of the existing nanosatellites. For instance, Fig. 1 shows an example of selecting the structural parameter of the nanosatellite for orbit altitude H = 380 km (the planned altitude of the group of the nanosatellites in the international project QB50) for given values $\alpha^*_{\max} = 20\,\deg$, $p^* = 0.95$, $\alpha_0 = 0$, $\sigma = 0.05$ deg/s. As it can be seen, the value of the structural parameter of the nanosatellite for the given motion should be $d \geq 0.13 m/kg$. Fig. 2 shows an example of setting the requirements for the initial transverse angular velocity of nanosatellite SamSat QB50 for the given values H = 380 km, $\alpha^*_{\max} = 30\,\deg$, $p^* = 0.95$, $\alpha_0 = 0$. As it can be seen, in order that the nanosatellite SamSat QB50 will commit the given motion, it is necessary to satisfy the requirement: $\sigma \leq 0.05$ deg/s.

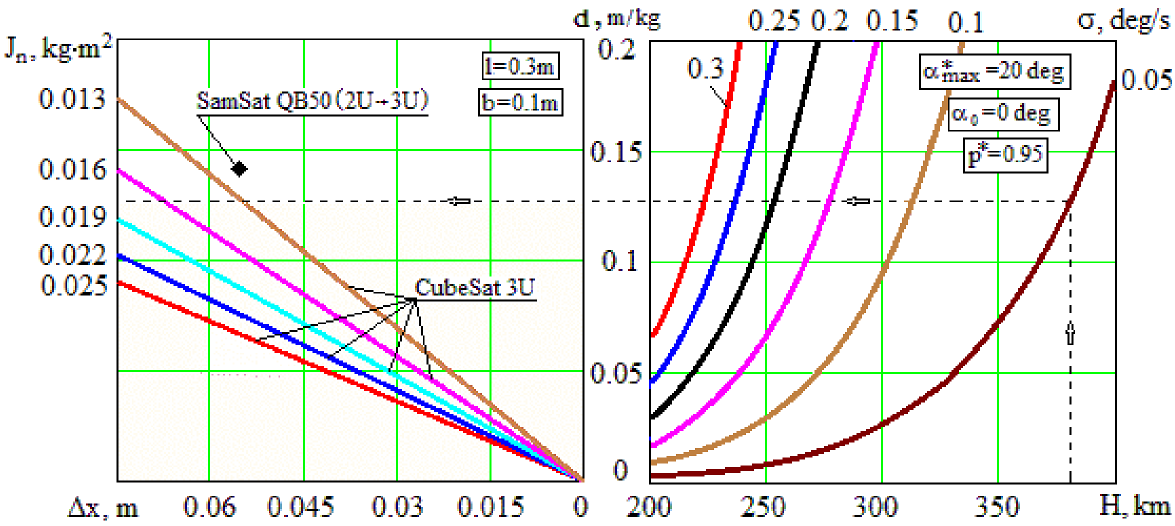

Fig. 1. The nomogram for selecting the structural parameter of the nanosatellite depending on the altitude H and the parameter value σ at $\alpha^*_{\max} = 20$ deg, $p^* = 0.95$, $\alpha_0 = 0$.

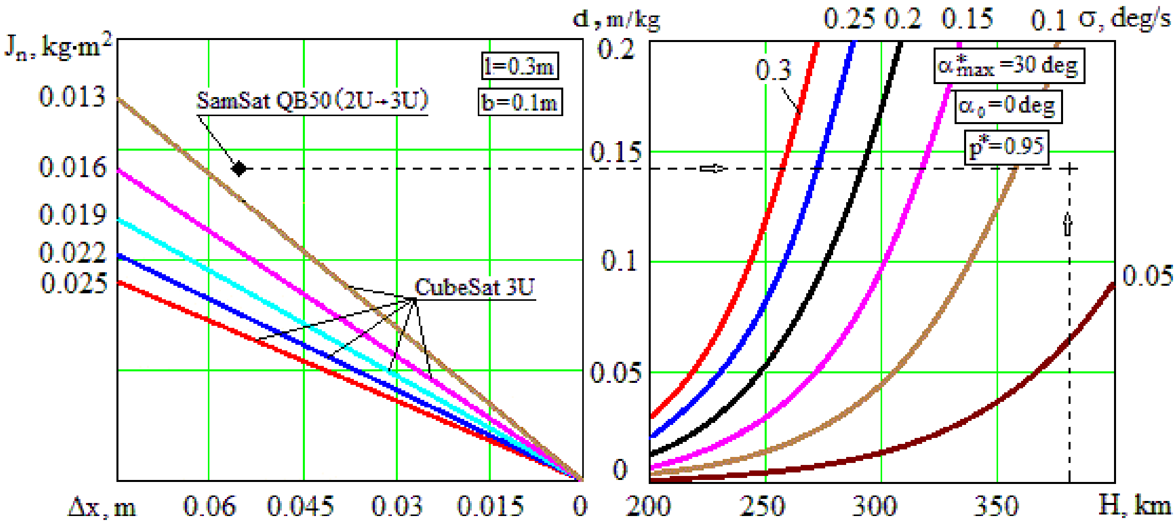

Fig. 2. The nomogram for selecting the structural parameter of the nanosatellite depending on the altitude H and the parameter value σ at $\alpha^*_{\max} = 30$ deg, $p^* = 0.95$, $\alpha_0 = 0$.

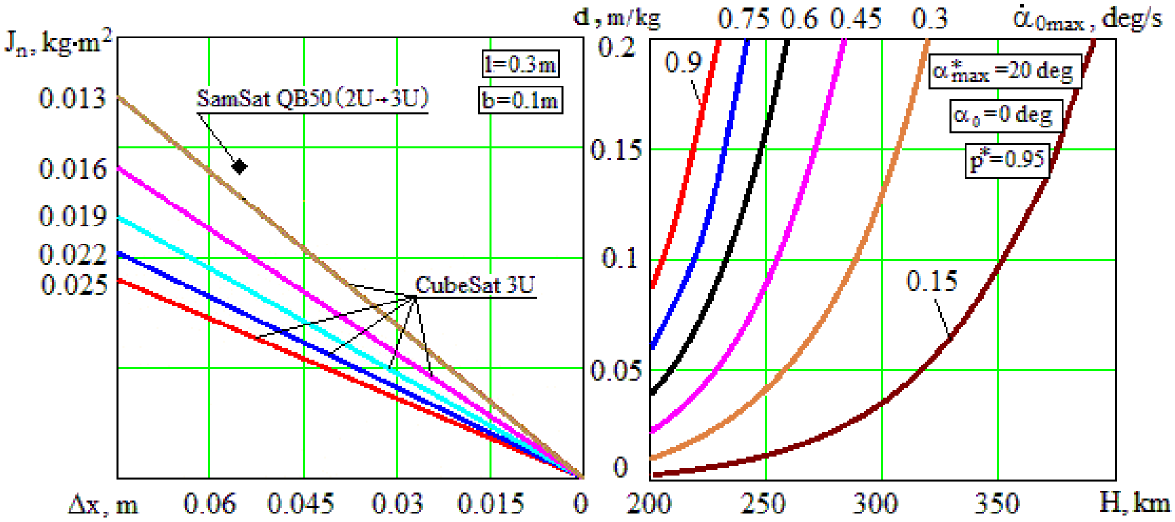

Fig. 3. The nomogram for selecting the structural parameter of the nanosatellite depending on the altitude H and the value of the right border of interval for initial transverse angular velocity $\dot{\alpha}_{0\,\max}$ at $\alpha^*_{\max} = 20$ deg , $p^* = 0.95$, $\alpha_0 = 0$.

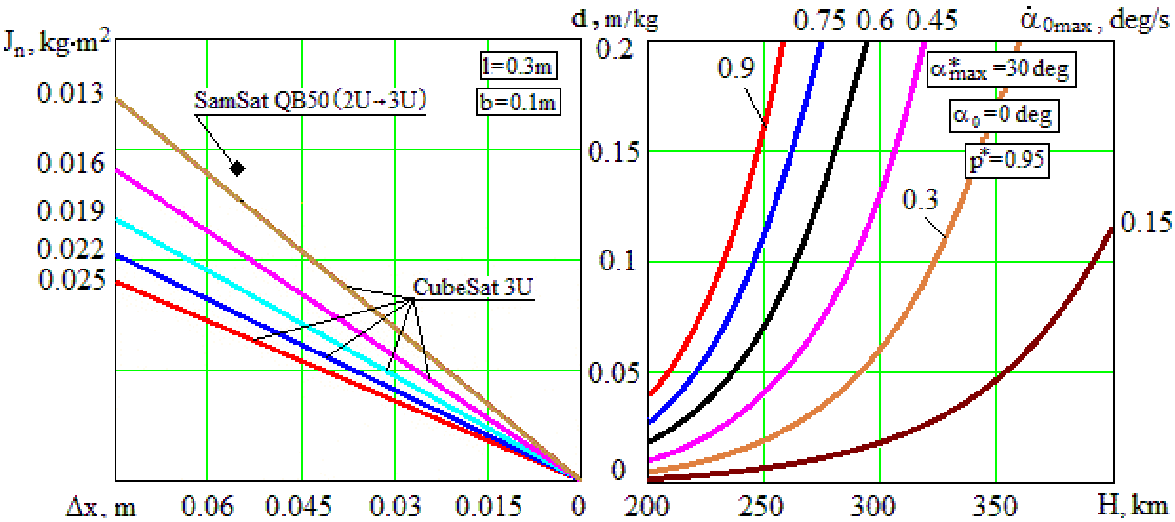

Fig. 4. The nomogram for selecting the structural parameter of the nanosatellite depending on the altitude H and the value of the right border of interval for initial transverse angular velocity $\dot{\alpha}_{0\,\max}$ at $\alpha^*_{\max} = 30$ deg , $p^* = 0.95$, $\alpha_0 = 0$.

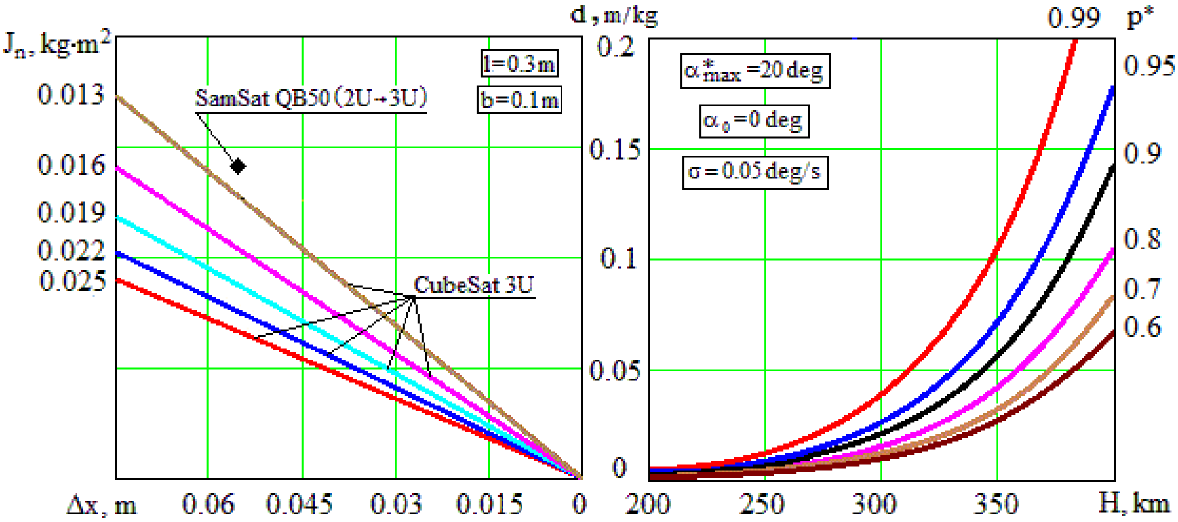

Fig. 5. The nomogram for selecting the structural parameter of the nanosatellite depending on the altitude H and the value of given probability p^* at $\alpha^*_{\max} = 20$ deg , $\alpha_0 = 0$, $\sigma = 0.05$ deg/s.

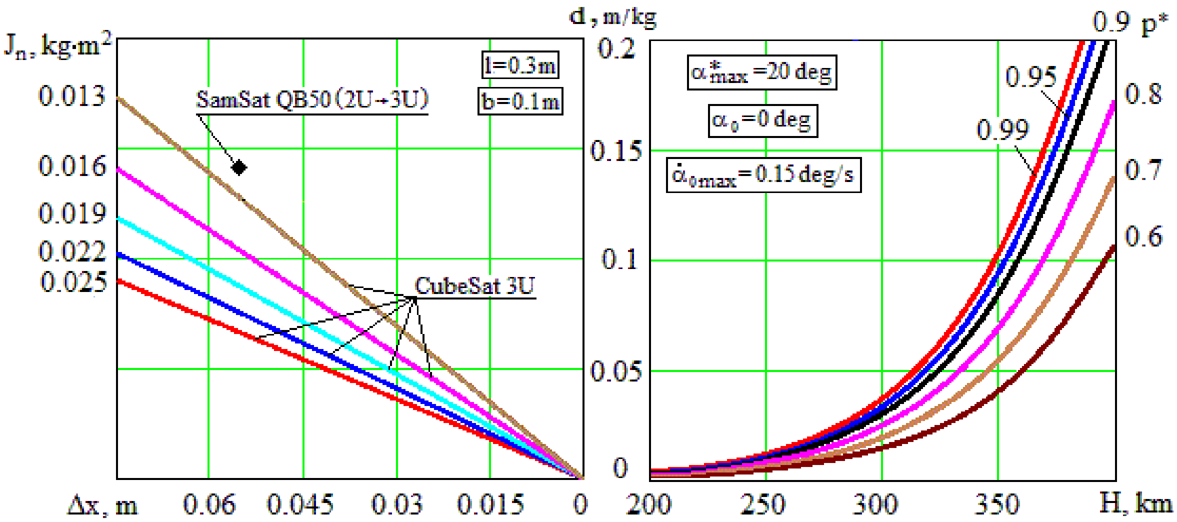

Fig. 6. The nomogram for selecting the structural parameter of the nanosatellite depending on the altitude H and the value of given probability p^* at $\alpha^*_{\max} = 20$ deg , $\alpha_0 = 0$, $\dot{\alpha}_{0\max} = 0.15$ deg/s.

3. Conclusion

Thus, the formulas for selecting the design parameters (static stability factor, length, longitudinal moment of inertia) of the aerodynamically stabilized nanosatellite of the CubeSat standard are obtained. At the low circular orbits the design parameters provides deviation of the longitudinal axis of the nanosatellite from the center of mass velocity vector that is less than acceptable with a given probability on a given height at the known errors of angular velocity for separation system.

The results reflect the general conditions for the realization of the mission of the aerodynamically stabilized nanosatellite. The obtained results are used to create the SamSat-QB50 nanosatellite as the part of the international project QB50.

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